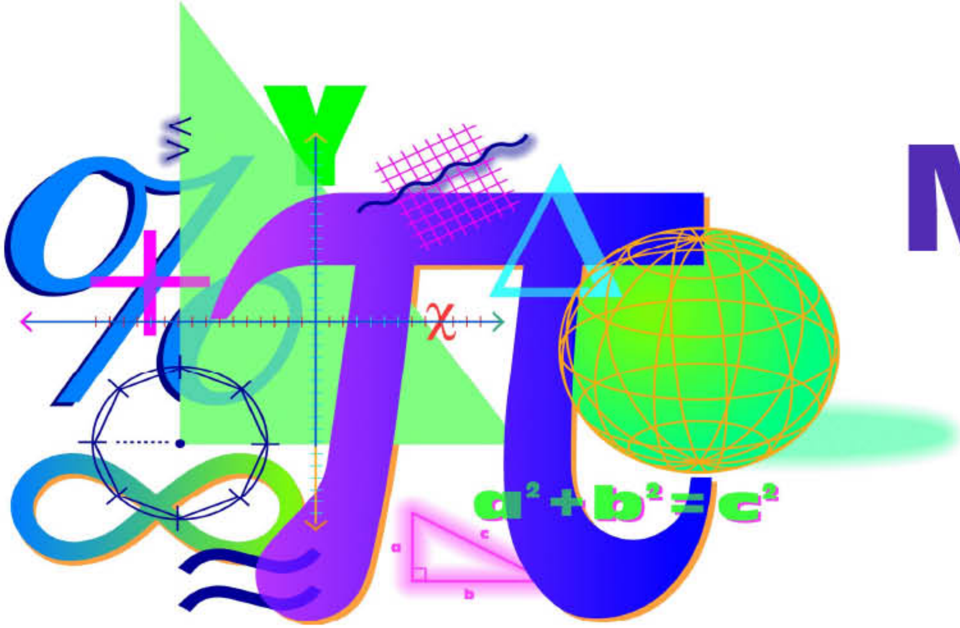

Mathematics
Standards of Learning

Curriculum Framework 2009

Grade 7

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The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia *Mathematics Standards of Learning* Curriculum Framework 2009
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into three columns: Understanding the Standard; Essential Understandings; and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes background information for the teacher (K-8). It contains content that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning. In Grades 6-8, these essential understandings are presented as questions to facilitate teacher planning.

Essential Knowledge and Skills
Each standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct a more advanced understanding of mathematics through active learning experiences;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students in the middle grades focus on mastering rational numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to most middle school mathematics topics.
- Students develop an understanding of integers and rational numbers by using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation. Flexible thinking about rational number representations is encouraged when students solve problems.
- Students develop an understanding of the properties of operations on real numbers through experiences with rational numbers and by applying the order of operations.
- Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

7.1 The student will
a) investigate and describe the concept of negative exponents for powers of ten;
b) determine scientific notation for numbers greater than zero;
c) compare and order fractions, decimals, percents and numbers written in scientific notation;
d) determine square roots; and
e) identify and describe absolute value for rational numbers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Negative exponents for powers of 10 are used to represent numbers between 0 and 1. (e.g., $10^{-3} = \frac{1}{10^3} = 0.001$). • Negative exponents for powers of 10 can be investigated through patterns such as: $10^2 = 100$ $10^1 = 10$ $10^0 = 1$ $10^{-1} = \frac{1}{10^1} = \frac{1}{10} = 0.1$ • A number followed by a percent symbol (%) is equivalent to that number with a denominator of 100 (e.g., $\frac{3}{5} = \frac{60}{100} = 0.60 = 60\%$). • Scientific notation is used to represent very large or very small numbers. • A number written in scientific notation is the product of two factors — a decimal greater than or equal to 1 but less than 10, and a power of 10 (e.g., $3.1 \times 10^5 = 310{,}000$ and $2.85 \times 10^{-4} = 0.000285$).	• When should scientific notation be used? Scientific notation should be used whenever the situation calls for use of very large or very small numbers. • How are fractions, decimals and percents related? Any rational number can be represented in fraction, decimal and percent form. • What does a negative exponent mean when the base is 10? A base of 10 raised to a negative exponent represents a number between 0 and 1. • How is taking a square root different from squaring a number? Squaring a number and taking a square root are inverse operations. • Why is the absolute value of a number positive? The absolute value of a number represents distance from zero on a number line regardless of direction. Distance is positive.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize powers of 10 with negative exponents by examining patterns. • Write a power of 10 with a negative exponent in fraction and decimal form. • Write a number greater than 0 in scientific notation. • Recognize a number greater than 0 in scientific notation. • Compare and determine equivalent relationships between numbers larger than 0 written in scientific notation. • Represent a number in fraction, decimal, and percent forms. • Compare, order, and determine equivalent relationships among fractions, decimals, and percents. Decimals are limited to the thousandths place, and percents are limited to the tenths place. Ordering is limited to no more than 4 numbers. • Order no more than 3 numbers greater than 0 written in scientific notation. • Determine the square root of a perfect square less than or equal to 400.

7.1 The student will

a) investigate and describe the concept of negative exponents for powers of ten;
b) determine scientific notation for numbers greater than zero;
c) compare and order fractions, decimals, percents and numbers written in scientific notation;
d) determine square roots; and
e) identify and describe absolute value for rational numbers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Equivalent relationships among fractions, decimals, and percents can be determined by using manipulatives (e.g., fraction bars, Base-10 blocks, fraction circles, graph paper, number lines and calculators). • A square root of a number is a number which, when multiplied by itself, produces the given number (e.g., $\sqrt{121}$ is 11 since 11 x 11 = 121). • The square root of a number can be represented geometrically as the length of a side of the square. • The absolute value of a number is the distance from 0 on the number line regardless of direction. (e.g., $\left\lvert \frac{-1}{2} \right\rvert = \frac{1}{2}$).		• Demonstrate absolute value using a number line. • Determine the absolute value of a rational number. • Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle to solve practical problems.† †Revised March 2011

7.2 The student will describe and represent arithmetic and geometric sequences using variable expressions.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• In the numeric pattern of an arithmetic sequence, students must determine the difference, called the *common difference*, between each succeeding number in order to determine what is added to each previous number to obtain the next number. • In geometric sequences, students must determine what each number is multiplied by in order to obtain the next number in the geometric sequence. This multiplier is called the *common ratio*. Sample geometric sequences include – 2, 4, 8, 16, 32, …; 1, 5, 25, 125, 625, …; and 80, 20, 5, 1.25, …. • A variable expression can be written to express the relationship between two consecutive terms of a sequence - If n represents a number in the sequence 3, 6, 9, 12…, the next term in the sequence can be determined using the variable expression $n + 3$. - If n represents a number in the sequence 1, 5, 25, 125…, the next term in the sequence can be determined by using the variable expression $5n$.	• When are variable expressions used? Variable expressions can express the relationship between two consecutive terms in a sequence.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Analyze arithmetic and geometric sequences to discover a variety of patterns. • Identify the common difference in an arithmetic sequence. • Identify the common ratio in a geometric sequence. • Given an arithmetic or geometric sequence, write a variable expression to describe the relationship between two consecutive terms in the sequence.

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Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring meaning to why procedures work and make sense.
- Students develop and refine estimation strategies and develop an understanding of when to use algorithms and when to use calculators. Students learn when exact answers are appropriate and when, as in many life experiences, estimates are equally appropriate.
- Students learn to make sense of the mathematical tools they use by making valid judgments of the reasonableness of answers.
- Students reinforce skills with operations with whole numbers, fractions, and decimals through problem solving and application activities.

7.3 The student will
a) model addition, subtraction, multiplication and division of integers; and
b) add, subtract, multiply, and divide integers.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The set of integers is the set of whole numbers and their opposites (e.g., … –3, –2, –1, 0, 1, 2, 3, …). • Integers are used in practical situations, such as temperature changes (above/below zero), balance in a checking account (deposits/withdrawals), and changes in altitude (above/below sea level). • Concrete experiences in formulating rules for adding and subtracting integers should be explored by examining patterns using calculators, along a number line and using manipulatives, such as two-color counters, or by using algebra tiles. • Concrete experiences in formulating rules for multiplying and dividing integers should be explored by examining patterns with calculators, along a number line and using manipulatives, such as two-color counters, or by using algebra tiles.	• The sums, differences, products and quotients of integers are either positive, zero, or negative. How can this be demonstrated? This can be demonstrated through the use of patterns and models.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model addition, subtraction, multiplication and division of integers using pictorial representations of concrete manipulatives. • Add, subtract, multiply, and divide integers. • Simplify numerical expressions involving addition, subtraction, multiplication and division of integers using order of operations. • Solve practical problems involving addition, subtraction, multiplication, and division with integers.

7.4 The student will solve single-step and multistep practical problems, using proportional reasoning.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A proportion is a statement of equality between two ratios. • A proportion can be written as $\frac{a}{b} = \frac{c}{d}$, *a*:*b* = *c*:*d*, or *a* is to *b* as *c* is to *d*. • A proportion can be solved by finding the product of the means and the product of the extremes. For example, in the proportion *a*:*b* = *c*:*d*, *a* and *d* are the extremes and *b* and *c* are the means. If values are substituted for *a*, *b*, *c*, and *d* such as 5:12 = 10:24, then the product of extremes (5 × 24) is equal to the product of the means (12 × 10). • In a proportional situation, both quantities increase or decrease together. • In a proportional situation, two quantities increase multiplicatively. Both are multiplied by the same factor. • A proportion can be solved by finding equivalent fractions. • A rate is a ratio that compares two quantities measured in different units. A unit rate is a rate with a denominator of 1. Examples of rates include miles/hour and revolutions/minute. • Proportions are used in everyday contexts, such as speed, recipe conversions, scale drawings, map reading, reducing and enlarging, comparison shopping, and monetary conversions.	• What makes two quantities proportional? Two quantities are proportional when one quantity is a constant multiple of the other.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write proportions that represent equivalent relationships between two sets. • Solve a proportion to find a missing term. • Apply proportions to convert units of measurement between the U.S. Customary System and the metric system. Calculators may be used. • Apply proportions to solve practical problems, including scale drawings. Scale factors shall have denominators no greater than 12 and decimals no less than tenths**.** Calculators may be used. • Using 10% as a benchmark, mentally compute 5%, 10%, 15%, or 20% in a practical situation such as tips, tax and discounts. • Solve problems involving tips, tax, and discounts. Limit problems to only one percent computation per problem.

7.4 The student will solve single-step and multistep practical problems, using proportional reasoning.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• Proportions can be used to convert between measurement systems. For example: if 2 inches is about 5 cm, how many inches are in 16 cm? – $\frac{2 inches}{x} = \frac{5 cm}{16 cm}$ • A percent is a special ratio in which the denominator is 100. • Proportions can be used to represent percent problems as follows: – $\frac{percent}{100} = \frac{part}{whole}$		

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Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop the measurement skills that provide a natural context and connection among many mathematics concepts. Estimation skills are developed in determining length, weight/mass, liquid volume/capacity, and angle measure. Measurement is an essential part of mathematical explorations throughout the school year.
- Students continue to focus on experiences in which they measure objects physically and develop a deep understanding of the concepts and processes of measurement. Physical experiences in measuring various objects and quantities promote the long-term retention and understanding of measurement. Actual measurement activities are used to determine length, weight/mass, and liquid volume/capacity.
- Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, pyramids, and cones.

7.5 The student will
a) describe volume and surface area of cylinders;
b) solve practical problems involving the volume and surface area of rectangular prisms and cylinders; and
c) describe how changing one measured attribute of a rectangular prism affects its volume and surface area.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The area of a rectangle is computed by multiplying the lengths of two adjacent sides. • The area of a circle is computed by squaring the radius and multiplying that product by π ($A = \pi r^2$, where $\pi \approx 3.14$ or $\frac{22}{7}$). • A rectangular prism can be represented on a flat surface as a net that contains six rectangles — two that have measures of the length and width of the base, two others that have measures of the length and height, and two others that have measures of the width and height. The surface area of a rectangular prism is the sum of the areas of all six faces ($SA = 2lw + 2lh + 2wh$). • A cylinder can be represented on a flat surface as a net that contains two circles (bases for the cylinder) and one rectangular region whose length is the circumference of the circular base and whose width is the height of the cylinder. The surface area of the cylinder is the area of the two circles and the rectangle ($SA = 2\pi r^2 + 2\pi rh$). • The volume of a rectangular prism is computed by multiplying the area of the base, *B*, (length times width) by the height of the prism ($V = lwh = Bh$). • The volume of a cylinder is computed by multiplying the area of the base, *B*, (πr^2) by the height of the cylinder ($V = \pi r^2 h = Bh$).	• How are volume and surface area related? Volume is a measure of the amount a container holds while surface area is the sum of the areas of the surfaces on the container. • How does the volume of a rectangular prism change when one of the attributes is increased? There is a direct relationship between the volume of a rectangular prism increasing when the length of one of the attributes of the prism is changed by a scale factor.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine if a practical problem involving a rectangular prism or cylinder represents the application of volume or surface area. • Find the surface area of a rectangular prism. • Solve practical problems that require finding the surface area of a rectangular prism. • Find the surface area of a cylinder. • Solve practical problems that require finding the surface area of a cylinder. • Find the volume of a rectangular prism. • Solve practical problems that require finding the volume of a rectangular prism. • Find the volume of a cylinder. • Solve practical problems that require finding the volume of a cylinder. • Describe how the volume of a rectangular prism is affected when one measured attribute is multiplied by a scale factor. Problems will be limited to changing attributes by scale factors only. • Describe how the surface area of a rectangular prism is affected when one measured attribute is multiplied by a scale factor. Problems will be limited to changing attributes by scale factors only.

7.5 The student will

a) describe volume and surface area of cylinders;

b) solve practical problems involving the volume and surface area of rectangular prisms and cylinders; and

c) describe how changing one measured attribute of a rectangular prism affects its volume and surface area.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• There is a direct relationship between changing one measured attribute of a rectangular prism by a scale factor and its volume. For example, doubling the length of a prism will double its volume. This direct relationship does not hold true for surface area.		

7.6 The student will determine whether plane figures – quadrilaterals and triangles – are similar and write proportions to express the relationships between corresponding sides of similar figures.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Two polygons are similar if corresponding (matching) angles are congruent and the lengths of corresponding sides are proportional. • Congruent polygons have the same size and shape. • Congruent polygons are similar polygons for which the ratio of the corresponding sides is 1:1. • Similarity statements can be used to determine corresponding parts of similar figures such as: $\Delta ABC \sim \Delta DEF$ $\angle A$ corresponds to $\angle D$ $\overline{AB}$ corresponds to $\overline{DE}$ • The traditional notation for marking congruent angles is to use a curve on each angle. Denote which angles are congruent with the same number of curved lines. For example, if $\angle$A congruent to $\angle$B, then both angles will be marked with the same number of curved lines. • Congruent sides are denoted with the same number of hatch marks on each congruent side. For example, a side on a polygon with 2 hatch marks is congruent to the side with 2 hatch marks on a congruent polygon.	• How do polygons that are similar compare to polygons that are congruent? Congruent polygons have the same size and shape. Similar polygons have the same shape, and corresponding angles between the similar figures are congruent. However, the lengths of the corresponding sides are proportional. All congruent polygons are considered similar with the ratio of the corresponding sides being 1:1.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify corresponding sides and corresponding and congruent angles of similar figures using the traditional notation of curved lines for the angles. • Write proportions to express the relationships between the lengths of corresponding sides of similar figures. • Determine if quadrilaterals or triangles are similar by examining congruence of corresponding angles and proportionality of corresponding sides. • Given two similar figures, write similarity statements using symbols such as $\Delta ABC \sim \Delta DEF$, $\angle A$ corresponds to $\angle D$, and $\overline{AB}$ corresponds to $\overline{DE}$.

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- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students expand the informal experiences they have had with geometry in the elementary grades and develop a solid foundation for the exploration of geometry in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.
- Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, miras, and geometry software provides experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.
- Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.
- Students explore and develop an understanding of the Pythagorean Theorem. Mastery of the use of the Pythagorean Theorem has far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades 5 and 6 and fully attain it before taking algebra.)
- **Level 4: Deduction.** Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)

7.7 The student will compare and contrast the following quadrilaterals based on properties: parallelogram, rectangle, square, rhombus, and trapezoid.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• A quadrilateral is a closed plane (two-dimensional) figure with four sides that are line segments. • A parallelogram is a quadrilateral whose opposite sides are parallel and opposite angles are congruent. • A rectangle is a parallelogram with four right angles. The diagonals of a rectangle are the same length and bisect each other. • A square is a rectangle with four congruent sides whose diagonals are perpendicular. A square is a rhombus with four right angles. • A rhombus is a parallelogram with four congruent sides whose diagonals bisect each other and intersect at right angles. • A trapezoid is a quadrilateral with exactly one pair of parallel sides. • A trapezoid with congruent, nonparallel sides is called an *isosceles trapezoid*. • Quadrilaterals can be sorted according to common attributes, using a variety of materials. • A chart, graphic organizer, or Venn diagram can be made to organize quadrilaterals according to attributes such as sides and/or angles.	• Why can some quadrilaterals be classified in more than one category? Every quadrilateral in a subset has all of the defining attributes of the subset. For example, if a quadrilateral is a rhombus, it has all the attributes of a rhombus. However, if that rhombus also has the additional property of 4 right angles, then that rhombus is also a square.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast attributes of the following quadrilaterals: parallelogram, rectangle, square, rhombus, and trapezoid. • Identify the classification(s) to which a quadrilateral belongs, using deductive reasoning and inference.

7.8 The student, given a polygon in the coordinate plane, will represent transformations (reflections, dilations, rotations, and translations) by graphing in the coordinate plane.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A rotation of a geometric figure is a turn of the figure around a fixed point. The point may or may not be on the figure. The fixed point is called the *center of rotation*. • A translation of a geometric figure is a slide of the figure in which all the points on the figure move the same distance in the same direction. • A reflection is a transformation that reflects a figure across a line in the plane. • A dilation of a geometric figure is a transformation that changes the size of a figure by scale factor to create a similar figure. • The image of a polygon is the resulting polygon after the transformation. The preimage is the polygon before the transformation. • A transformation of preimage point A can be denoted as the image A' (read as "A prime").	• How does the transformation of a figure affect the size, shape and position of that figure? Translations, rotations and reflections do not change the size or shape of a figure. A dilation of a figure and the original figure are similar. Reflections, translations and rotations usually change the position of the figure.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the coordinates of the image of a right triangle or rectangle that has been translated either vertically, horizontally, or a combination of a vertical and horizontal translation. • Identify the coordinates of the image of a right triangle or rectangle that has been rotated 90° or 180° about the origin. • Identify the coordinates of the image of a right triangle or a rectangle that has been reflected over the x- or y-axis. • Identify the coordinates of a right triangle or rectangle that has been dilated. The center of the dilation will be the origin. • Sketch the image of a right triangle or rectangle translated vertically or horizontally. • Sketch the image of a right triangle or rectangle that has been rotated 90° or 180° about the origin. • Sketch the image of a right triangle or rectangle that has been reflected over the x- or y-axis. • Sketch the image of a dilation of a right triangle or rectangle limited to a scale factor of $\frac{1}{4}$, $\frac{1}{2}$, 2, 3 or 4.

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- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students develop an awareness of the power of data analysis and probability by building on their natural curiosity about data and making predictions.
- Students explore methods of data collection and use technology to represent data with various types of graphs. They learn that different types of graphs represent different types of data effectively. They use measures of center and dispersion to analyze and interpret data.
- Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability.
- Students explore experimental and theoretical probability through experiments and simulations by using concrete, active learning activities.

7.9 The student will investigate and describe the difference between the experimental probability and theoretical probability of an event.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Theoretical probability of an event is the expected probability and can be found with a formula. • Theoretical probability of an event = $\frac{\text{number of possible favorable outcomes}}{\text{total number of possible outcomes}}$ • The experimental probability of an event is determined by carrying out a simulation or an experiment. • The experimental probability = $\frac{\text{number of times desired outcomes occur}}{\text{number of trials in the experiment}}$ • In experimental probability, as the number of trials increases, the experimental probability gets closer to the theoretical probability (Law of Large Numbers).	• What is the difference between the theoretical and experimental probability of an event? Theoretical probability of an event is the expected probability and can be found with a formula. The experimental probability of an event is determined by carrying out a simulation or an experiment. In experimental probability, as the number of trials increases, the experimental probability gets closer to the theoretical probability.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the theoretical probability of an event. • Determine the experimental probability of an event. • Describe changes in the experimental probability as the number of trials increases. • Investigate and describe the difference between the probability of an event found through experiment or simulation versus the theoretical probability of that same event.

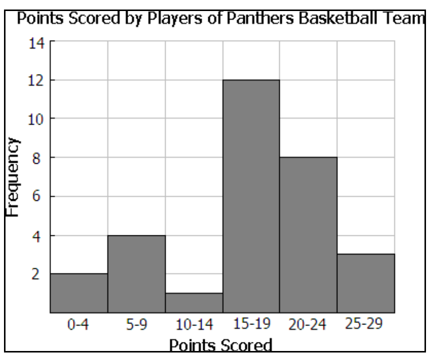
7.10 The student will determine the probability of compound events, using the Fundamental (Basic) Counting Principle.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The Fundamental (Basic) Counting Principle is a computational procedure to determine the number of possible outcomes of several events. It is the product of the number of outcomes for each event that can be chosen individually (e.g., the possible outcomes or outfits of four shirts, two pants, and three shoes is $4 \cdot 2 \cdot 3$ or 24). • Tree diagrams are used to illustrate possible outcomes of events. They can be used to support the Fundamental (Basic) Counting Principle. • A compound event combines two or more simple events. For example, a bag contains 4 red, 3 green and 2 blue marbles. What is the probability of selecting a green and then a blue marble?	• What is the Fundamental (Basic) Counting Principle? The Fundamental (Basic) Counting Principle is a computational procedure used to determine the number of possible outcomes of several events. • What is the role of the Fundamental (Basic) Counting Principle in determining the probability of compound events? The Fundamental (Basic) Counting Principle is used to determine the number of outcomes of several events. It is the product of the number of outcomes for each event that can be chosen individually.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compute the number of possible outcomes by using the Fundamental (Basic) Counting Principle. • Determine the probability of a compound event containing no more than 2 events.

7.11 The student, given data in a practical situation, will
a) construct and analyze histograms; and
b) compare and contrast histograms with other types of graphs presenting information from the same data set.

UNDERSTANDING THE STANDARD
(Background Information for Instructor Use Only)

- All graphs tell a story and include a title and labels that describe the data.
- A histogram is a form of bar graph in which the categories are consecutive and equal intervals. The length or height of each bar is determined by the number of data elements frequency falling into a particular interval.



- A frequency distribution shows how often an item, a number, or range of numbers occurs. It can be used to construct a histogram.

STUDENTS WHO READ GARFIELD				
Age Group	Tally	Frequency	Cumulative Frequency	
7-10	𝍸 II	7	7	
11-14	𝍸 II	7	14	← 7 + 7
15-18	III	3	17	← 14 + 3
19-22	III	3	20	← 17 + 3

ESSENTIAL UNDERSTANDINGS

- What types of data are most appropriate to display in a histogram?

  Numerical data that can be characterized using consecutive intervals are best displayed in a histogram.

ESSENTIAL KNOWLEDGE AND SKILLS

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Collect, analyze, display, and interpret a data set using histograms. For collection and display of raw data, limit the data to 20 items.
- Determine patterns and relationships within data sets (e.g., trends).
- Make inferences, conjectures, and predictions based on analysis of a set of data.
- Compare and contrast histograms with line plots, circle graphs, and stem-and-leaf plots presenting information from the same data set.

7.11 The student, given data in a practical situation, will
a) construct and analyze histograms; and
b) compare and contrast histograms with other types of graphs presenting information from the same data set.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Comparisons, predictions and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions. • The information displayed in different graphs may be examined to determine how data are or are not related, ascertaining differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or “what could happen if” (inference).		

In the middle grades, the focus of mathematics learning is to

- build on students' concrete reasoning experiences developed in the elementary grades;
- construct through active learning experiences a more advanced understanding of mathematics;
- develop deep mathematical understandings required for success in abstract learning experiences; and
- apply mathematics as a tool in solving practical problems.

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands.

- Students extend their knowledge of patterns developed in the elementary grades and through practical experiences by investigating and describing functional relationships.
- Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable, term, coefficient, exponent, expression, equation, inequality*, *domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades.
- Students learn to solve equations by using concrete materials. They expand their skills from one-step to two-step equations and inequalities.
- Students learn to represent relations by using ordered pairs, tables, rules, and graphs. Graphing in the coordinate plane linear equations in two variables is a focus of the study of functions.

7.12 The student will represent relationships with tables, graphs, rules, and words.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Rules that relate elements in two sets can be represented by word sentences, equations, tables of values, graphs, or illustrated pictorially. • A relation is any set of ordered pairs. For each first member, there may be many second members. • A function is a relation in which there is one and only one second member for each first member. • As a table of values, a function has a unique value assigned to the second variable for each value of the first variable. • As a graph, a function is any curve (including straight lines) such that any vertical line would pass through the curve only once. • Some relations are functions; all functions are relations.	• What are the different ways to represent the relationship between two sets of numbers? Rules that relate elements in two sets can be represented by word sentences, equations, tables of values, graphs or illustrated pictorially.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe and represent relations and functions, using tables, graphs, rules, and words. Given one representation, students will be able to represent the relation in another form.

7.13 The student will

a) write verbal expressions as algebraic expressions and sentences as equations and vice versa; and

b) evaluate algebraic expressions for given replacement values of the variables.

UNDERSTANDING THE STANDARD (Teacher Notes)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• An expression is a name for a number. • An expression that contains a variable is a variable expression. • An expression that contains only numbers is a numerical expression. • A verbal expression is a word phrase (e.g., “the sum of two consecutive integers”). • A verbal sentence is a complete word statement (e.g., “The sum of two consecutive integers is five.”). • An algebraic expression is a variable expression that contains at least one variable (e.g., $2x – 5$). • An algebraic equation is a mathematical statement that says that two expressions are equal (e.g., $2x + 1 = 5$). • To evaluate an algebraic expression, substitute a given replacement value for a variable and apply the order of operations. For example, if a = 3 and b = -2 then 5a + b can be evaluated as: 5(3) + (-2) = 15 + (-2) = 13.	• How can algebraic expressions and equations be written? Word phrases and sentences can be used to represent algebraic expressions and equations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write verbal expressions as algebraic expressions. Expressions will be limited to no more than 2 operations. • Write verbal sentences as algebraic equations. Equations will contain no more than 1 variable term. • Translate algebraic expressions and equations to verbal expressions and sentences. Expressions will be limited to no more than 2 operations. • Identify examples of expressions and equations. • Apply the order of operations to evaluate expressions for given replacement values of the variables. Limit the number of replacements to no more than 3 per expression.

7.14 The student will
a) solve one- and two-step linear equations in one variable; and
b) solve practical problems requiring the solution of one- and two-step linear equations.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• An equation is a mathematical sentence that states that two expressions are equal. • A one-step equation is defined as an equation that requires the use of one operation to solve (e.g., $x + 3 = -4$). • The inverse operation for addition is subtraction, and the inverse operation for multiplication is division. • A two-step equation is defined as an equation that requires the use of two operations to solve (e.g., $2x + 1 = -5$; $-5 = 2x + 1$; $\frac{x-7}{3} = 4$).	• When solving an equation, why is it important to perform identical operations on each side of the equal sign? An operation that is performed on one side of an equation must be performed on the other side to maintain equality.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent and demonstrate steps for solving one- and two-step equations in one variable using concrete materials, pictorial representations and algebraic sentences. • Solve one- and two-step linear equations in one variable. • Solve practical problems that require the solution of a one- or two-step linear equation.

7.15 The student will
a) solve one-step inequalities in one variable; and
b) graph solutions to inequalities on the number line.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A one-step inequality is defined as an inequality that requires the use of one operation to solve (e.g., $x - 4 > 9$). • The inverse operation for addition is subtraction, and the inverse operation for multiplication is division. • When both expressions of an inequality are multiplied or divided by a negative number, the inequality symbol reverses (e.g., $-3x < 15$ is equivalent to $x > -5$). • Solutions to inequalities can be represented using a number line.	• How are the procedures for solving equations and inequalities the same? The procedures are the same except for the case when an inequality is multiplied or divided on both sides by a negative number. Then the inequality sign is changed from less than to greater than, or greater than to less than. • How is the solution to an inequality different from that of a linear equation? In an inequality, there can be more than one value for the variable that makes the inequality true.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent and demonstrate steps in solving inequalities in one variable, using concrete materials, pictorial representations, and algebraic sentences. • Graph solutions to inequalities on the number line. • Identify a numerical value that satisfies the inequality.

7.16 The student will apply the following properties of operations with real numbers:

a) the commutative and associative properties for addition and multiplication;
b) the distributive property;
c) the additive and multiplicative identity properties;
d) the additive and multiplicative inverse properties; and
e) the multiplicative property of zero.

UNDERSTANDING THE STANDARD **(Background Information for Instructor Use Only)**	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The commutative property for addition states that changing the order of the addends does not change the sum (e.g., 5 + 4 = 4 + 5). • The commutative property for multiplication states that changing the order of the factors does not change the product (e.g., 5 · 4 = 4 · 5). • The associative property of addition states that regrouping the addends does not change the sum [e.g., 5 + (4 + 3) = (5 + 4) + 3]. • The associative property of multiplication states that regrouping the factors does not change the product [e.g., 5 · (4 · 3) = (5 · 4) · 3]. • Subtraction and division are neither commutative nor associative. • The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number [e.g., 5 · (3 + 7) = (5 · 3) + (5 · 7), or 5 · (3 – 7) = (5 · 3) – (5 · 7)]. • Identity elements are numbers that combine with other numbers without changing the other numbers. The additive identity is zero (0). The multiplicative identity is one (1). There are no identity elements for subtraction and division.	• Why is it important to apply properties of operations when simplifying expressions? Using the properties of operations with real numbers helps with understanding mathematical relationships.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify properties of operations used in simplifying expressions. • Apply the properties of operations to simplify expressions.

7.16 The student will apply the following properties of operations with real numbers:

a) the commutative and associative properties for addition and multiplication;

b) the distributive property;

c) the additive and multiplicative identity properties;

d) the additive and multiplicative inverse properties; and

e) the multiplicative property of zero.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	**ESSENTIAL UNDERSTANDINGS**	**ESSENTIAL KNOWLEDGE AND SKILLS**
• The additive identity property states that the sum of any real number and zero is equal to the given real number (e.g., 5 + 0 = 5). • The multiplicative identity property states that the product of any real number and one is equal to the given real number (e.g., $8 \cdot 1 = 8$). • Inverses are numbers that combine with other numbers and result in identity elements [e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$]. • The additive inverse property states that the sum of a number and its additive inverse always equals zero [e.g., 5 + (–5) = 0]. • The multiplicative inverse property states that the product of a number and its multiplicative inverse (or reciprocal) always equals one (e.g., $4 \cdot \frac{1}{4} = 1$). • Zero has no multiplicative inverse. • The multiplicative property of zero states that the product of any real number and zero is zero. • Division by zero is not a possible arithmetic operation. Division by zero is undefined.		